Three-phase Windings Theory

A theory based on the Crisci List

Javier Alejandro Gallo

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1 Basic definitions

1.1 Basic structure: the three-phase winding

Definition 1 (three-phase winding)**.** *A three-phase winding (or $W3$) is a tuple $w = (poles, span, slots, branches, phase\_groups)$ that satisfies the following axioms:*

axiom 1 $0 < poles, span, slots, branches, phase\_groups \in \mathbb{Z}$

axiom 2 $poles = 2k, k \in \mathbb{Z}$

axiom 3 $3 \cdot poles \leq slots$

axiom 4 .

1. $span \leq 4 \cdot q(w)$
2. $span < \dfrac{slots}{2} + 1$

axiom 5 $slots = 3m, m \in \mathbb{Z}$

where $q(w) = \dfrac{slot}{3 \cdot poles}$

1.2 Useful definitions

Definition 2. *Let $w = (poles, span, slots, branches, phase\_groups) \in W3$.*

$$span_d(w) = \frac{slots}{poles} + 1 \tag{1}$$

$$Qint(w) \equiv q(w) \in \mathbb{Z} \tag{2}$$

$$Qpar(w) \equiv \exists k \in \mathbb{Z} : q(w) = 2k \tag{3}$$

$$Serie(w) \equiv branches = 1 \tag{4}$$

$$Bp(w) \equiv poles = 2 \tag{5}$$

We call *diametral span* to the value defined in 1. Windings that satisfy 5 are called *bipolar windings.*

1.3 Types of three-phase windings

Definition 3 (homologous three-phase winding)**.** *Let* $w =$
$= (poles, span, slots, branches, phase\_groups) \in W3$*. We'll say that* w *is a homologous three-phase winding, denoted by* $Homolog(w)$*, when:*

property 1 $phase\_groups = \dfrac{poles}{2}$

property 2 $poles = 2 \cdot branches \cdot k, k \in \mathbb{Z}$

property 3 $3 \cdot q(w) \leq span$
(note that this property is equivalent to $span_d(w) - 1 \leq span$).

Definition 4 (number of groups)**.** *Let* $w =$
$(poles, span, slots, branches, phase\_groups) \in W3$*. The number of groups of* w *is given by* $groups(w)$*. The function is defined by the following expression:*

$$
\begin{aligned}
&groups : W3 \to \mathbb{Z}_{\geq 0} \\
&groups(w) = 3 \cdot phase\_groups
\end{aligned}
$$

Definition 5 (single layer winding)**.** *Let* $w =$
$= (poles, span, slots, branches, phase\_groups) \in W3$*. We'll call* w *a single layer three-phase winding, denoted by Simple(w), iff it satisfies the following properties:*

property 1 $\neg Homolog(w) \implies q(w) \in \mathbb{Z}$

property 2 $poles = 2 \implies q(w) \in \mathbb{Z}$

property 3 .

$$
\begin{aligned}
&\neg Homolog(w) \implies 2 \cdot q(w) \leq span \\
&Homolog(w) \wedge \frac{slots}{2 \cdot groups(w)} \in \mathbb{Z} \implies span = span_d(w)
\end{aligned}
$$

Definition 6 (interleaved three-phase winding)**.** *Let* $w =$
$= (poles, span, slots, branches, phase\_groups) \in W3$*. We'll say that* w *is a interleaved three-phase winding, denoted by* $Inter(w)$*, when:*

property 1 $phase\_groups = poles$

property 2 $poles = branches \cdot k, k \in \mathbb{Z}$

property 3 $\dfrac{span_d(w)}{2} \leq span$

property 4 $Simple(w) \implies q(w) \in \mathbb{Z}$

Definition 7 (special winding)**.** *A* $w \in W3$ *is a special winding iff* $\neg Homolog(w) \wedge \neg Inter(w)$*.*

Definition 8 (phase turns)**.** *Let* $w = (poles, span, slots, branches, phase\_groups) \in W3$*. The number of phase turns of* w *is given by* $phase\_turns(w)$*, as defined below:*

$$phase\_turns(w) = \begin{cases} slots/6 & \textit{if } Simple(w) \\ slots/3 & \textit{otherwise} \end{cases}$$

Definition 9 (group turns)**.** *Let* $w = (poles, span, slots, branches, phase\_groups) \in W3$*. The number of group turns of* w *is given by* $group\_turns(w)$*, as defined below:*

$$group\_turns(w) = \frac{phase\_turns(w)}{phase\_groups}$$

Definition 10 (regular three-phase winding)**.** *A* $w \in W3$ *is a regular three-phase winding, denoted by* $Regular(w)$*, iff* $group\_turns(w) \in \mathbb{Z}$*. It is irregular iff it is not regular.*

2 Theorems

Theorem 1 (bipolarity)**.** *Let* $w = (poles, span, slots, branches, phase\_groups) \in W3$*, with* $Bp(w)$*, then:*

$$span < span_d(w)$$

Proof. First, let's see that by definition, $span_d(w) = \frac{slots}{poles} + 1 = \frac{slots}{2} + 1$. But $w \in W3$, so w satisfies **axiom 4** stating that $span < \frac{slots}{2} + 1 = span_d(w)$. □

Theorem 2 (group turns)**.** *Let* $w \in W3$*.*

(a) If $Simple(w)$*, then:*

(a1) $Homolog(w) \implies group\_turns(w) = q(w)$

(a2) $\neg Homolog(w) \implies group\_turns(w) = \dfrac{q(w)}{2}$

(b) If $\neg Simple(w)$*, then:*

(b1) $Homolog(w) \implies group\_turns(w) = 2 \cdot q(w)$

(b2) $\neg Homolog(w) \implies group\_turns(w) = q(w)$

Proof. Let's suppose that $w = (poles, span, slots, branches, phase\_groups)$. We will only show (a1), the rest of cases are similar.

In this case we know that $Simple(w)$ and $Homolog(w)$, so:

$$\begin{aligned}
group_turns(w) = & & \text{(definition of } group_turns\text{)}\\
&= \frac{phase_turns(w)}{phase_groups} & \text{(definition of } phase_turns\text{)}\\
&= \frac{slots}{6 \cdot phase_groups} & \text{(}\textbf{property 1}\text{ of homologous winding)}\\
&= \frac{slots}{6 \cdot \frac{poles}{2}} & \\
&= \frac{slots}{3 \cdot poles} & \text{(definition of q)}\\
&= q(w)
\end{aligned}$$

□

Theorem 3 (diametral span)**.** *Let* $w = (poles, span, slots, branches, phase\_groups) \in W3$*, with* $Regular(w)$*,* $Homolog(w)$ *and* $Simple(w)$*, then:*

$$span = span_d(w).$$

Proof. As $Regular(w)$, we know that $group\_turns(w) \in \mathbb{Z}$. But then by definition of $group\_turns$ it follows that $phase\_turns(w)$ is a multiple of $phase\_groups$. $Simple(w)$ as we know, so by definition of $phase\_turns$ we see that $\frac{slots}{6}$ is a multiple of $phase\_groups$. Therefore, $slots$ must be a multiple of $6 \cdot phase\_groups$. By definition of $groups$, $6 \cdot phase\_groups = 2 \cdot groups(w)$, then $slots$ is a multiple of $2 \cdot groups(w)$. Thus, $\frac{slots}{2 \cdot groups(w)} \in \mathbb{Z}$.

Now, we know that $Homolog(w)$ and that $\frac{slots}{2 \cdot groups(w)} \in \mathbb{Z}$. We also know by hypothesis that $Simple(w)$. Therefore, by **property 3** of $Simple$, we conclude that $span = span_d(w)$. □

Theorem 4 (polarity)**.** *Let* $w = (poles, span, slots, branches, phase\_groups) \in W3$*. Then:*

$$\neg(Homolog(w) \wedge Inter(w))$$

This means that w *could be an homologous winding or an interleaved winding (or none), but not both at the same time.*

Proof. If we assume that $Homolog(w)$, then by **property 1** of homologous windings we know that $phase\_groups = \frac{poles}{2}$. But then $phase\_groups \neq poles$ because $poles > 0$, by **axiom 1** of three-phase windings. Thus, it can't be that $Inter(w)$ because in that case **property 1** of interleaved windings would be broken.

If we assume that $Inter(w)$, the proof procedure is similar. □